## NASA-TM-X-1303

DZ

NASA TM X-1303

NASA TECHNICAL MEMORANDUM



NASA TM X-1303

•

DISTRIBUTION STATEMENT A
Approved for Public Release
Distribution Unlimited

NONISENTROPIC NONSTEADY LIQUID FLOW WITH CENTRIFUGAL, GRAVITATIONAL, AND DISSIPATIVE FORCES

Reproduced From Best Available Copy

by Fred S. Sidransky

Lewis Research Center Cleveland, Ohio LOVELACE FOUNDATION
DOUGLENT LINKSMY

20011009 09

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION . WASHINGTON, D. C. . OCTOBER 1966

NASA TM X-1303

# NONISENTROPIC NONSTEADY LIQUID FLOW WITH CENTRIFUGAL, GRAVITATIONAL, AND DISSIPATIVE FORCES

By Fred S. Sidransky

Lewis Research Center Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

# NONISENTROPIC NONSTEADY LIQUID FLOW WITH CENTRIFUGAL, GRAVITATIONAL, AND DISSIPATIVE FORCES

by Fred S. Sidransky

Lewis Research Center

#### **SUMMARY**

The method of characteristics is used to present general compatibility relations for nonsteady liquid flow or water-hammer theory which permit the investigation of the dynamics of the flow under diverse conditions; that is, for a liquid flowing in a variable area conduit affected by heat addition, by gravity, by friction forces, and by centrifugal forces such as in a mixed-flow radial vane pump. The compatibility equations are presented in finite difference form, and numerical-graphical methods for their solution are indicated.

#### INTRODUCTION

In rocket engine and Rankine cycle systems, the analysis of nonsteady liquid flow is often complicated by the transmission of the fluid in a conduit with a gradually varying cross-sectional area. In addition, the fluid may be affected by heat addition and a number of forces such as gravity, dissipative, and centrifugal (e. g., as in a pump). Hence, a general method is required which will allow the investigation of fluid transients under such diverse conditions.

Analyses for nonsteady liquid flow have been presented in a number of papers for particular conditions; that is, for a tapered line (ref. 1), for friction (ref. 2), and for a pump as a lumped component in a conduit (ref. 3).

In this report the theory of nonsteady liquid flow as developed in reference 4 will be extended so that the dynamics of a liquid in a conduit with a gradually varying cross-sectional area, undergoing heat addition, and affected by the previously mentioned forces may be analyzed. Applications will be outlined for nonsteady liquid flow with heat addition, for tapered conduits with gravity and dissipative forces, and for the internal dynamics of a mixed flow pump with radial vanes.

#### **ANALYSIS**

#### Derivation of Fundamental Compatibility Relations

Continuity. - The continuity equation (ref. 5) is given by

$$\frac{\partial}{\partial t} (\rho A) + \frac{\partial}{\partial \xi} (\rho u A) = 0$$
 (1)

and if  $\frac{\partial \mathbf{A}}{\partial t} = \mathbf{0}$ , equation (1) may be expanded to

$$A \frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho}{\partial \xi} u + \frac{\partial u}{\partial \xi} \rho\right) A + \frac{\partial A}{\partial \xi} \rho u = 0$$
 (2)

The term  $\frac{\partial \mathbf{A}}{\partial t}$ , a function of the elasticity of the conduit, is omitted since it is usually negligibly small compared to the variation of area with distance along the conduit  $\frac{\partial \mathbf{A}}{\partial \xi}$ . If equation (2) is divided by  $\rho \mathbf{A}$ , then

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} + \frac{1}{\rho} u \frac{\partial \rho}{\partial \xi} + \frac{\partial u}{\partial \xi} + u \frac{\partial (\ln A)}{\partial \xi} = 0$$
 (3a)

and since

$$P = f(\rho, s) \tag{3b}$$

equation (3) may be expressed by

$$\frac{1}{\rho} \left( \frac{\partial \rho}{\partial \mathbf{P}} \frac{\partial \mathbf{P}}{\partial t} + \frac{\partial \rho}{\partial \mathbf{s}} \frac{\partial \mathbf{s}}{\partial t} \right) + \frac{1}{\rho} \left( \frac{\partial \rho}{\partial \mathbf{P}} \frac{\partial \mathbf{P}}{\partial \xi} + \frac{\partial \rho}{\partial \mathbf{s}} \frac{\partial \mathbf{s}}{\partial \xi} \right) \mathbf{u} + \frac{\partial \mathbf{u}}{\partial \xi} + \mathbf{u} \frac{\partial (\ln \mathbf{A})}{\partial \xi} = \mathbf{0}$$
(4)

Assuming that  $\frac{\partial \rho}{\partial s}$  is negligibly small for a liquid and using the definition for the acoustic velocity, namely,

$$\left(\frac{\partial \rho}{\partial \mathbf{P}}\right)_{\mathbf{S}} = \frac{1}{\mathbf{a}^2} \tag{5}$$

result in

$$\frac{1}{\rho a^2} \frac{\partial P}{\partial t} + \frac{1}{\rho a^2} \frac{\partial P}{\partial \xi} u + \frac{\partial u}{\partial \xi} + u \frac{\partial (\ln A)}{\partial \xi} = 0$$
 (6)

Inasmuch as the pressure P is expressed in force per unit area and the density  $\rho$  in mass per unit volume and the head in the units of pressure divided by the specific weight of the fluid in the Earth's gravitational field, the following relation may be deduced

$$\frac{P - P_0}{\rho} = g_c H \tag{7}$$

where  $P_0$  is the datum pressure, a constant. In view of equation (7) and the assumption that the variation of the density has a negligibly small effect on the head (i.e.,  $\rho$  = const.), equation (6) may be presented by

$$\frac{g_{\mathbf{c}}}{\mathbf{a}^{2}} \frac{\partial \mathbf{H}}{\partial t} + \frac{g_{\mathbf{c}}}{\mathbf{a}^{2}} \mathbf{u} \frac{\partial \mathbf{H}}{\partial \xi} + \frac{\partial \mathbf{u}}{\partial \xi} + \mathbf{u} \frac{\partial (\ln \mathbf{A})}{\partial \xi} = 0$$
 (8)

Multiplying by  $a^2/g_c$  and transposing some terms result in

$$\frac{\partial \mathbf{H}}{\partial \mathbf{t}} + \mathbf{u} \frac{\partial \mathbf{H}}{\partial \xi} + \frac{\mathbf{a}^2}{\mathbf{g}_c} \frac{\partial \mathbf{u}}{\partial \xi} = -\frac{\mathbf{u}\mathbf{a}^2}{\mathbf{g}_c} \frac{\partial (\ln \mathbf{A})}{\partial \xi}$$
(9)

Momentum. - Consider the momentum equation given by (ref. 5)

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u} \frac{\partial \mathbf{u}}{\partial \xi} = -\frac{1}{\rho} \frac{\partial \mathbf{P}}{\partial \xi} + \mathbf{f}$$
 (10)

Multiplying equation (10) by  $a/g_{c}$  and considering equation (7) give

$$\frac{\mathbf{a}}{\mathbf{g_c}} \frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \frac{\mathbf{a}}{\mathbf{g_c}} \mathbf{u} \frac{\partial \mathbf{u}}{\partial \xi} + \mathbf{a} \frac{\partial \mathbf{H}}{\partial \xi} = \frac{\mathbf{a}}{\mathbf{g_c}} \mathbf{f}$$
 (11)

Subtracting equation (11) from equation (9) yields

$$\frac{\partial}{\partial t} \left( H - \frac{a}{g_c} u \right) + (u - a) \frac{\partial H}{\partial \xi} - \frac{a}{g_c} \left( u \frac{\partial u}{\partial \xi} - a \frac{\partial u}{\partial \xi} \right) = -\frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi} - \frac{a}{g_c} f$$
 (12)

if  $a/g_{\rm c}$  is a constant. By further algebraic manipulation, equation (12) can be presented in the form

$$\frac{\partial}{\partial t} \left( H - \frac{a}{g_c} u \right) + (u - a) \frac{\partial}{\partial \xi} \left( H - \frac{a}{g_c} u \right) = -\frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi} - \frac{a}{g_c} f$$
 (13)

Adding equations (9) and (11) yields another expression, namely,

$$\frac{\partial}{\partial t} \left( H + \frac{a}{g_c} u \right) + (u + a) \frac{\partial}{\partial \xi} \left( H + \frac{a}{g_c} u \right) = -\frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi} + \frac{a}{g_c} f$$
 (14)

Now let

$$H + \frac{a}{g_c} u = C_x \tag{15}$$

and

$$H - \frac{a}{g_c} u = C_y$$
 (16)

and define two directional derivatives as follows:

$$\frac{\delta^{+}}{\delta t} = \frac{\partial}{\partial t} + (u + a) \frac{\partial}{\partial \xi}$$
 (17)

and

$$\frac{\delta^{-}}{\delta t} = \frac{\partial}{\partial t} + (u - a) \frac{\partial}{\partial \xi}$$
 (18)

With the help of equations (15) to (18), equations (13) and (14) may be expressed by

$$\frac{\delta^{-}C_{y}}{\delta t} = -\frac{ua^{2}}{g_{c}} \frac{\partial (\ln A)}{\partial \xi} - \frac{a}{g_{c}} f$$
 (19)

$$\frac{\delta^{+}C_{x}}{\delta t} = -\frac{ua^{2}}{g_{c}} \frac{\partial (\ln A)}{\partial \xi} + \frac{a}{g_{c}} f$$
 (20)

In the terminology of the method of characteristics (ref. 5, vol. 1, appendix A), equations (19) and (20) can be considered as right running and left running compatibility equations respectively, and their characteristic directions in the time-distance plane are

$$\frac{d\xi}{dt} = u - a \tag{21}$$

along the right running characteristic and

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = \mathrm{u} + \mathrm{a} \tag{22}$$

along the left running characteristic. The similarity of these results and those of reference 4 is evident.

If the flow velocity is very small compared to the acoustic velocity, equations (21) and (22) become simply

$$\frac{\mathrm{d}\xi}{\mathrm{dt}} = -\mathbf{a} \tag{23}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = \mathbf{a} \tag{24}$$

If the two terms appearing to the right of equations (19) and (20) are omitted or set equal to zero, then

$$\frac{\delta^{+}C_{y}}{\delta t} = 0 \tag{25a}$$

and

$$C_{y} = H - \frac{a}{g_{c}} u \tag{25b}$$

is the right running compatibility equation and

$$C_{X} = H + \frac{a}{g_{c}} u \tag{26}$$

is the left running compatibility relation. These equations are the more familiar waterhammer relations.

<u>Energy</u>. - The energy equation gives rise to a third compatibility relation and characteristic direction. This may be seen by considering the energy equation in a conduit as expressed by (ref. 5)

$$q\rho A = \frac{\partial}{\partial t} \left[ \rho A \left( i + \frac{u^2}{2} \right) \right] + \frac{\partial}{\partial \xi} \left[ \rho u A \left( i + Pv + \frac{u^2}{2} \right) \right]$$
 (27)

which after some manipulation becomes

$$q\rho A = \left(i + \frac{u^2}{2}\right) \left[\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial \xi}\right] + \frac{\partial(u A P)}{\partial \xi} + \rho A \left(\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial \xi}\right) + \rho A \left[\frac{\partial}{\partial t} \left(\frac{u^2}{2}\right) + u \frac{\partial}{\partial \xi} \left(\frac{u^2}{2}\right)\right]$$
(28)

By the definition of the co-moving or substantial derivative, namely,

$$\frac{\mathbf{D}}{\mathbf{Dt}} = \frac{\partial}{\partial t} + \frac{\mathrm{d}\xi}{\mathrm{d}t} \frac{\partial}{\partial \xi}$$
 (29a)

or

$$\frac{\mathbf{D}}{\mathbf{D}t} = \frac{\partial}{\partial t} + \mathbf{u} \frac{\partial}{\partial \xi}$$
 (29b)

and, in view of equation (1), equation (28) may be presented by

$$\mathbf{P} \frac{\partial (\mathbf{u}\mathbf{A})}{\partial \xi} + \mathbf{u}\mathbf{A} \frac{\partial \mathbf{P}}{\partial \xi} + \rho \mathbf{A} \frac{\mathbf{D}\mathbf{i}}{\mathbf{D}\mathbf{t}} + \rho \mathbf{A}\mathbf{u} \frac{\mathbf{D}\mathbf{u}}{\mathbf{D}\mathbf{t}} = \mathbf{q}\rho \mathbf{A}$$
 (30)

From continuity, the following relation is obtained:

$$\frac{\partial (uA)}{\partial \xi} = A\rho \frac{D\left(\frac{1}{\rho}\right)}{Dt}$$
 (31)

where  $\frac{\partial \mathbf{A}}{\partial t}$  is omitted. Hence, instead of equation (30),

$$\rho AP \frac{D\left(\frac{1}{\rho}\right)}{Dt} + \rho Au \left(\frac{1}{\rho} \frac{\partial P}{\partial \xi} + \frac{Du}{Dt}\right) + \rho A \frac{Di}{Dt} = \rho Aq$$
 (32)

is obtained. In view of equation (10), equation (32) may be transformed to

$$\mathbf{P} \frac{\mathbf{D}\left(\frac{1}{\rho}\right)}{\mathbf{D}t} + \frac{\mathbf{D}\mathbf{i}}{\mathbf{D}t} + \mathbf{u}\mathbf{f} = \mathbf{q} \tag{33}$$

Since, from thermodynamics, it is known that

$$P \frac{D\left(\frac{1}{\rho}\right)}{Dt} + \frac{Di}{Dt} = T \frac{Ds}{Dt}$$
(34)

it follows that

Time, t

$$T \frac{Ds}{Dt} = q (35)$$

if the magnitude of the term uf, which represents in the negative sense the specific work

done by dissipative forces per unit time, is much smaller than the value of q (i.e., the heat addition is reversible). Equation (35) is the third compatibility equation; and in view of equations (29a) and (29b), the direction of the characteristic in the time-distance plane will be given by

Left running characteristic 
$$C_X$$
 Right running characteristic  $C_Y$  Particle path

Distance along conduit, &

Figure 1. - General characteristic net point for nonsteady liquid flow or water hammer.

$$\frac{d\xi}{dt} = u \tag{36}$$

Hence, in general, a net point in nonsteady liquid flow is defined by three characteristics, the projection of which on the time-distance plane is seen in figure 1.

Heat addition. - Equations (15) and (16) are two equations with two unknowns, H and u, since  $C_{\rm x}$  is a constant along the left running

characteristic and  $C_y$  is a constant along the right running characteristic, and the acoustic velocity a is also constant. If none of the parameters of equations (15) and (16) are affected by entropy change, which can be reasonably assumed in liquids, the contribution of heat addition to the dynamic analysis of liquids may be omitted. Yet it is desirable to estimate the temperature variation of a liquid as it proceeds, for example, through a boiler or heated pump inlet so as to predict the incipience of boiling. This may be accomplished by following a particle of fixed identity in the fluid as implied in equation (35). From thermodynamics, it is given that for constant volume reversible heating

$$T Ds = C_{v}DT (37)$$

Integrating equation (37) along the particle path (cf. fig. 1) results in

$$T_3 = T_4 e^{(s_3 - s_4)/C_v}$$
 (38)

and, consequently, by equation (35)

$$T_3 = T_4 e^{\psi(t_3 - t_4)/C_v}$$
 (39)

where

$$\psi = \frac{\mathbf{q}}{\mathbf{T}}$$

#### **Evaluation of the Specific Force**

The specific force f (or force per unit mass) appearing in the fundamental momentum equation (eq. (10)) is undefined. It may be equated to a specific gravity force, or a dissipative force, or a specific centrifugal force, or the sum of all these depending on the particular study to be made. In the following, the specific force f will be equated, first, to the sum of the specific gravity and specific dissipative forces and, then, in order to analyze the internal dynamics of a mixed-flow radial pump, to the sum of specific centrifugal and specific dissipative forces.

Gravity and dissipative forces. - If the gravity force per unit mass, or in other words the specific gravity force g, is opposite in direction to the assumed positive direction of the specific force f, the sum of the specific gravity and dissipative forces may be expressed by

$$f = -\frac{\lambda u^2}{2D} \frac{u}{|u|} - g \tag{40}$$

where  $\lambda$  is a friction coefficient and D is the hydraulic diameter of the duct. The factor u/|u| ensures that the dissipative force is opposite in direction to the flow velocity. By equation (40) the compatibility equations (eqs. (19) and (20)) become

$$\frac{\delta^{-}C_{y}}{\delta t} = -\frac{ua^{2}}{g_{c}}\frac{\partial(\ln A)}{\partial \xi} + \frac{a}{g_{c}}\left(\frac{\lambda u^{2}}{2D}\frac{u}{|u|} + g\right)$$
(41)

$$\frac{\delta^{+}C_{x}}{\delta t} = -\frac{ua^{2}}{g_{c}}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_{c}}\left(\frac{\lambda u^{2}}{2D}\frac{u}{|u|} + g\right)$$
(42)

or, in finite difference form,

$$C_{y_3} = C_{y_2} - \left[ \frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi} \right]_{23} (t_3 - t_2) + \left( \frac{a}{g_c} \frac{\lambda u^2}{2D} \frac{u}{|u|} + g \right)_{23} (t_3 - t_2)$$
(43)

$$C_{x_3} = C_{x_1} - \left[ \frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi} \right]_{13} (t_3 - t_1) - \left( \frac{a}{g_c} \frac{\lambda u^2}{2D} \frac{u}{|u|} + g \right)_{13} (t_3 - t_1)$$
(44)

and, for the corresponding characteristic directions,

$$\frac{\xi_3 - \xi_2}{t_3 - t_2} = -a_{23} \tag{45}$$

$$\frac{\xi_3 - \xi_1}{t_3 - t_1} = a_{13} \tag{46}$$

where the subscripts refer to net points in the characteristic network (cf. fig. 1). The subscripts 13 and 23 represent average values between net point 1 and 3 and net point 2 and 3, respectively. Clearly, equations (43) and (44) may be more easily evaluated if the variation of area with conduit length is expressed in the form

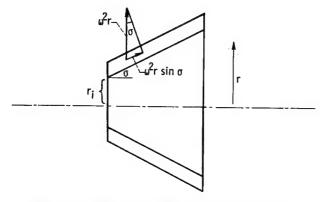


Figure 2. – Mixed-flow pump with radial vanes. Seeing that  $r-r_i=\xi\sin\sigma$ , it follows that  $f=\omega^2\xi\sin^2\sigma+\omega^2r_i\sin\sigma$ .

$$A = e^{h(\xi)}$$
 (47a)

or

$$\frac{\partial (\ln A)}{\partial \xi} = \frac{\partial h(\xi)}{\partial \xi}$$
 (47b)

where  $h(\xi)$  is some easily differentiable function of conduit length.

Pump dynamics. - The equation of motion for a mixed-flow pump with radial

vanes may be expressed by (cf. fig. 2)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{u}}{\partial \xi} - \left(\omega^2 \xi \sin^2 \sigma + \omega^2 \mathbf{r_i} \sin \sigma\right) = -\frac{1}{\rho} \frac{\partial \mathbf{P}}{\partial \xi}$$
 (48)

where u is the through-flow component of the flow velocity relative to the blade and  $\sigma$  is the angle the through-flow component makes with the axis of the pump. The terms  $\omega^2\xi\sin^2\sigma+\omega^2r_i\sin\sigma$  represent the centrifugal force per unit mass. If, in addition, friction forces are significant, then

$$f = \omega^2 \xi \sin^2 \sigma + \omega^2 r_i \sin \sigma - \frac{\lambda u^2}{2D} \frac{u}{|u|}$$

and equations (41) and (42) may be modified to

$$\frac{\delta^{-C}y}{\delta t} = -\frac{ua^{2}}{g_{c}}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_{c}}\left(\omega^{2}\xi \sin^{2}\sigma + \omega^{2}r_{i}\sin\sigma - \frac{\lambda u^{2}}{2D}\frac{u}{|u|}\right)$$
(49)

$$\frac{\delta^{+}C_{x}}{\delta t} = -\frac{ua^{2}}{g_{c}} \frac{\partial (\ln A)}{\partial \xi} + \frac{a}{g_{c}} \left( \omega^{2} \xi \sin^{2} \sigma + \omega^{2} r_{i} \sin \sigma - \frac{\lambda u^{2}}{2D} \frac{u}{|u|} \right)$$
 (50)

or in finite difference form to

$$C_{y_3} = C_{y_2} - \left[\frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi}\right]_{23} (t_3 - t_2) - \left[\frac{a}{g_c} \left(\omega^2 \xi \sin^2 \sigma + \omega^2 r_i \sin \sigma - \frac{\lambda u^2}{2D} \frac{u}{|u|}\right)\right]_{23} (t_3 - t_2)$$
(51)

$$C_{x_3} = C_{x_1} - \left[\frac{ua^2}{g_c} \frac{\partial (\ln A)}{\partial \xi}\right]_{13} (t_3 - t_1) + \left[\frac{a}{g_c} \left(\omega^2 \xi \sin^2 \sigma + \omega^2 r_i \sin^2 \sigma - \frac{\lambda u^2}{2D} \frac{u}{|u|}\right)\right]_{13} (t_3 - t_1)$$
(52)

If cavitation occurs in the pump (the head is less than the vapor head), a vapor region will appear in the characteristic network. The vapor head and the  $C_{\mathbf{x}}$  characteristic will define the flow into the cavitation zone, and the  $C_v$  characteristic will define the flow out. The time increment and past conditions will help to estimate the augmentation in the size of the cavitation zone. Further details on the analysis of the dynamics of cavitation regions by essentially the method of characteristics may be found in reference 6.

#### **CONCLUDING REMARKS**

General compatibility relations have been presented which will permit the analysis of the dynamics of a liquid under diverse conditions. In view of their complexity, a highspeed computer should be employed in their practical application to any particular problem.

Although an electromagnetic pump has not been considered in this memorandum, it is clear that in place of the centrifugal force per unit mass term in the compatibility equations appertaining to pump dynamics, an electromagnetic force per unit mass term may be employed.

The characteristic methods of this report may be applied, for example, to an approximate estimate of the effect of the through-flow channel of a pump on flow dynamics. Further, the techniques of this report may be used to predict the dynamic formation and collapse of cavitation regions in a heated pump inlet line as well as the incipience of twophase flow in a boiler under dynamic conditions.

Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio, June 8, 1966,

120-27-04-27-22.

### APPENDIX A

### SYMBOLS

A	area, sq ft	S	specific entropy, ft-lb/
a	acoustic velocity, ft/sec		(slug)( <sup>O</sup> R)
$C_{\mathbf{v}}$	specific heat at constant vol-	T	temperature, <sup>o</sup> R
	ume, ft-lb/(slug)( <sup>0</sup> R)	t	time, sec
$C_{\mathbf{x}}$	parameter (see eq. (15)), ft	u	flow velocity, ft/sec
Cy	parameter (see eq. (16)), ft	v	specific volume, cu ft/slug
D	hydraulic diameter, ft	λ	friction coefficient
f	specific force or force per	ξ	distance along conduit, ft
	unit mass, ft/sec <sup>2</sup>	ρ	density, slug/cu ft
g	specific gravity force, ft/sec <sup>2</sup>	σ	angle that the relative
$g_c$	acceleration due to Earth's gravitational field, $ft/sec^2$		through-flow component of velocity makes with pump
H	head, ft		axis
i	internal energy, ft-lb/slug	$\psi$	parameter, q/T
P	pressure, lb/sq ft	ω	angular speed, sec <sup>-1</sup>
q	heat added per unit mass per	Subscripts:	
	unit time, ft-lb/(slug)(sec)	0	datum
r	radial height (see fig. 2)	1,2,3,4	refers to net point locations
r <sub>i</sub>	inlet radial height (see fig. 2)		(see fig. 1)

#### REFERENCES

- 1. Paynter, H. M.; and Ezekiel, F. D.: Water Hammer in Nonuniform Pipes as an Example of Wave Propagation in Gradually Varying Media. ASME Trans., vol. 80, Oct. 1958, pp. 1585-1595.
- 2. Streeter, Victor L.; and Lai, Chinter: Water-Hammer Analysis Including Fluid Friction. Am. Soc. Civil Eng. Proc. (J. Hydraulic Division), vol. 88, no. HY3, May 1962, pp. 79-112.
- 3. Dorsch, Robert G.; Wood, Don J.; and Lightner, Charlene: Distributed Parameter Analysis of Pressure and Flow Disturbances in Rocket Propellant Feed Systems. NASA TN D-3529, 1966.
- 4. Sidransky, Fred; and Smith, Margaret Marie: Nonsteady Liquid and Gas Flow with Heat Addition and Shock Perturbations. NASA TN D-3684, 1966.
- 5. Shapiro, Ascher H.: The Dynamics and Thermodynamics of Compressible Fluid Flow. Ronald Press Co., 1954, vol. II, pp. 972-974.
- 6. Bergeron, L.: Water Hammer in Hydraulics and Wave Surges in Electricity. John Wiley and Sons, Inc., 1961.

"The aeronautical and space activities of the United States shall be conducted so as to contribute... to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

-NATIONAL AERONAUTICS AND SPACE ACT OF 1958

#### NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546